

Practical Rationality and its relationship with Mathematical Knowledge for Teaching

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A final thought

- On the matter of knowledge for teaching we need to capitalize on the fact that teachers work in institutions, not alone
- We need to think of teacher knowledge as distributed between the knowledge in the heads of individuals and the knowledge extant in institutional artifacts and customs

The notion of **teacher knowledge**

- Has **evolved from** the time **when** we thought of teacher knowledge in terms of **degrees** or **coursework**
- Now, major proponents of MKT are able to **specify bits** of teacher knowledge and identify how those bits **make a difference in practice.**
- However, the notion of teacher knowledge continues to emphasize the **individual ownership** of this knowledge
- And creates the impression that **instructional improvement** consists in improving **the individual teacher**, their knowledge and skill

We need to look at instructional improvement from a broader perspective

- We take for granted that increasing individual's mathematical knowledge for teaching is one lever for instructional improvement.
- Inspired by two recent key contributions to the question of other levers for improvement
 - Atul Gawande's The Checklist Manifesto
 - Anne Morris and Jim Hiebert's Educational Researcher piece
- We'd like to argue with those scholars that another lever for improvement issues from the notion that **organizations can be more knowledgeable**
- Organizations (or more specifically **instructional systems**) for example
 - A specific class for a specific group of kids
 - Sets of same courses offered to comparable kids
 - Course sequences offered to comparable kids
 - Course offerings by a mathematics department in a school or district
 - All the teachers of high school geometry of the country
 - All the cooperating teachers of a given teacher education program

In the rest of the talk

- We propose a **framework** to think about organizational knowledge in instruction as composed of individual and collective components
- We illustrate the **collective** side and connect to the question of improvement by suggesting how **intellectual resources** (such as checklists, forms, rubrics) and their **research-based development** can help make instructional systems more knowledgeable

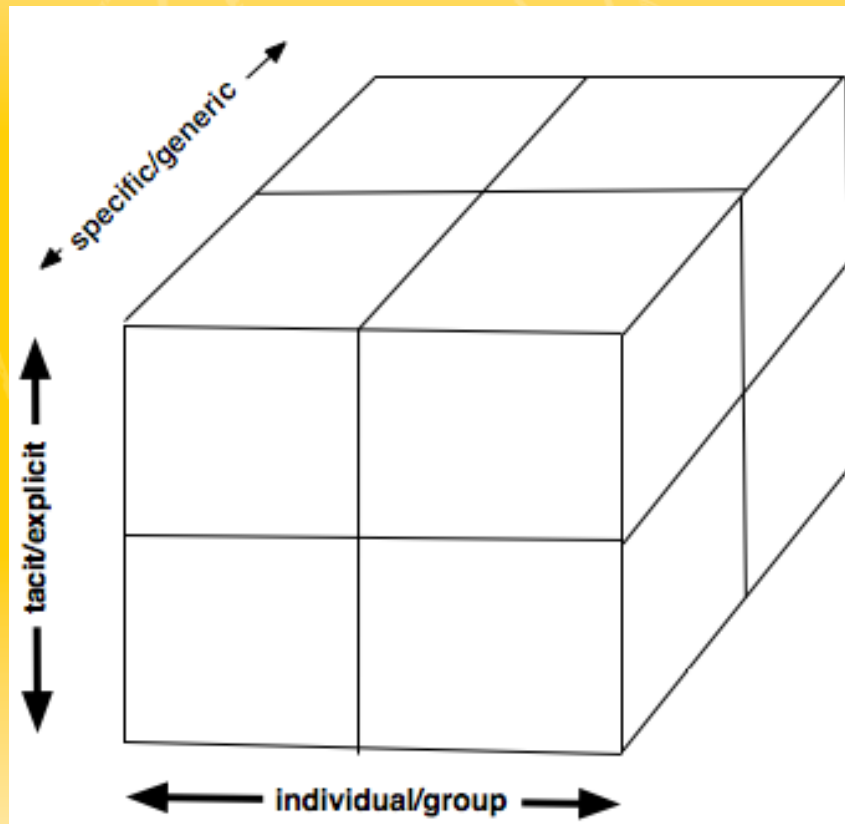
Cook and Brown (1999) proposed a framework for organizational knowledge

Examples

		where knowledge is held	
		individual	group
how knowledge is held	explicit	individually held and explicitly stated	collectively held and explicitly stated
	tacit	individually held and tacitly stated	collectively held and tacitly stated

What checks to make for algebraic expressions for the sides of an isosceles triangle	Scope and sequence of the high school mathematics curriculum
The skill to draw freehand diagrams that look accurate	Students expect teachers to know the answers to the questions they ask

An additional **distinction** between **generic** and **specific** knowledge could be useful



		Top Layer: Explicit	
Generic		Differences between higher and lower level cognitive demands in tasks	Policies and procedures related to the handling of special education children
Specific		What checks to make for algebraic expressions for the sides of an isosceles triangle	Scope and sequence of the high school mathematics curriculum
		Bottom Layer: Tacit	
Generic		How to question students without making them feel interrogated	Students expect teachers to know the answers to the questions they ask
Specific		How to draw freehand diagrams that look accurate	Teachers are expected to provide the given and the prove anytime students are expected to do a proof
		Individual	Group

The contributions of MKT and Practical Rationality

- The notion of mathematical knowledge for teaching
 - Is a contribution to the individual, specific stack
 - With elements that are tacit and elements that are explicit
- Our work under the heading “Practical Rationality” attempts to contribute to the collective, specific stack
 - Also with elements that are tacit and elements that are explicit

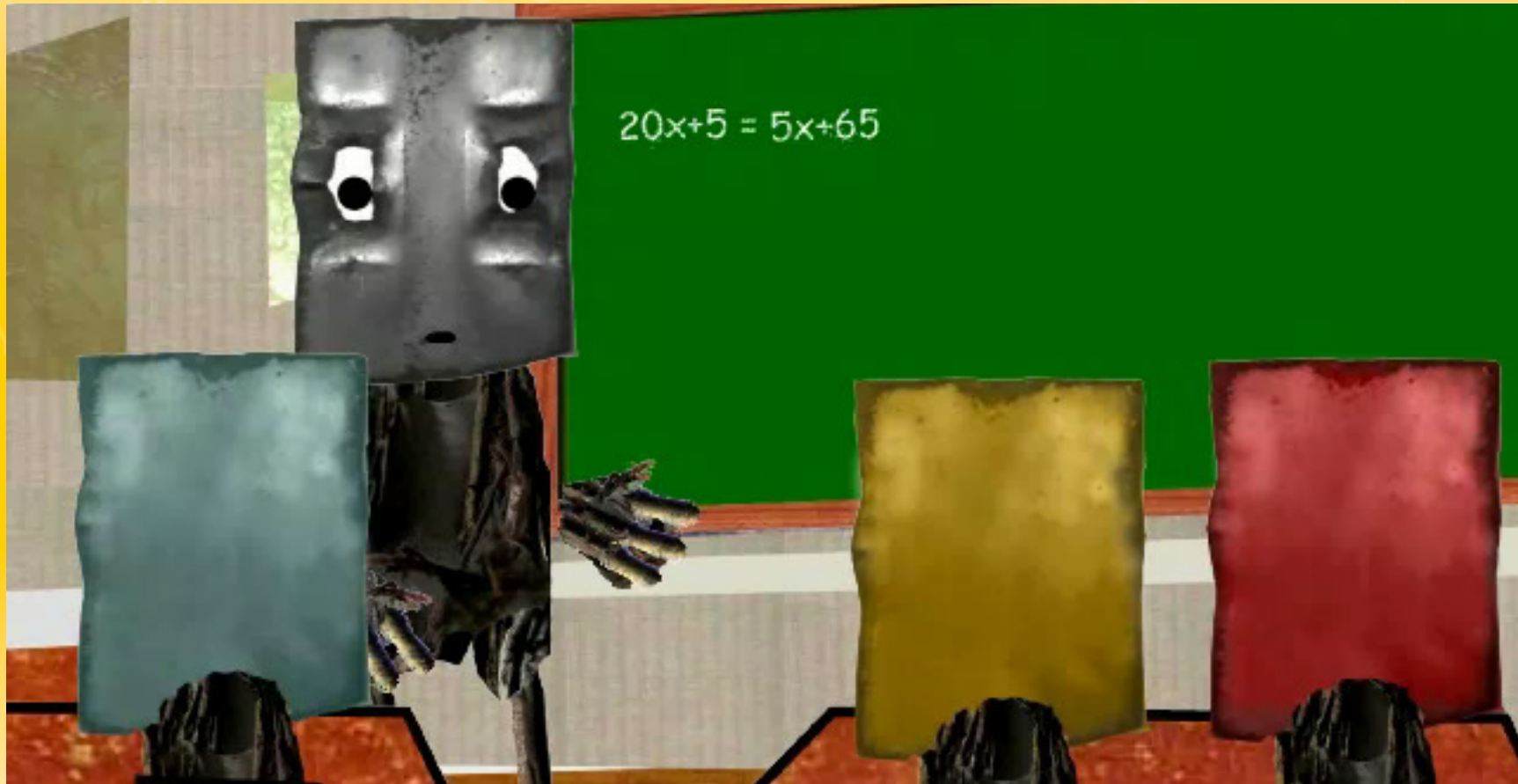
What checks to make for algebraic expressions for the sides of an isosceles triangle

The skill to draw freehand diagrams that look accurate

collectively held, mathematically specific, and explicitly stated

collectively held, mathematically specific, and tacitly stated

Would you use this equation?



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Instructional situations

- With the notion of “instructional situation” we refer to **subject specific genres of classroom work**, where students’ work on some particular kinds of **tasks** is taken by the teacher as indications of their possession of particular bits of **knowledge**.
- For example, their work on “solve $5x - 1 = 3x + 7$ ” is likely to exchange for a claim about students’ knowledge of methods for transforming an equation without transforming the solution set.
 - We call that exchange, that **instructional situation**, “**solving for x.**”
- Teachers as a group have special knowledge that they use in enacting that situation. For example **knowledge of the norms of instructional situations**.
- E.g., they would be unlikely to give a problem like “solve $5x + 65 = 20x + 5$ ”
- What is it that teachers know about instructional situations?
 - **As a group**, they know **the norms** of the instructional situations that cover the high school curriculum
 - **As a group**, they also know the possible consequences of **breaching** those norms.
 - Norms (in this context) are statements that describe who is supposed to do what and when in an instructional situation. Tacit expectations on the teacher and the student.
 - Knowledge of these norms is **tacit**, for the most part, **and held by the group**, for example distributed between teachers, exams and other artifacts, textbook writers, editors, and reviewers

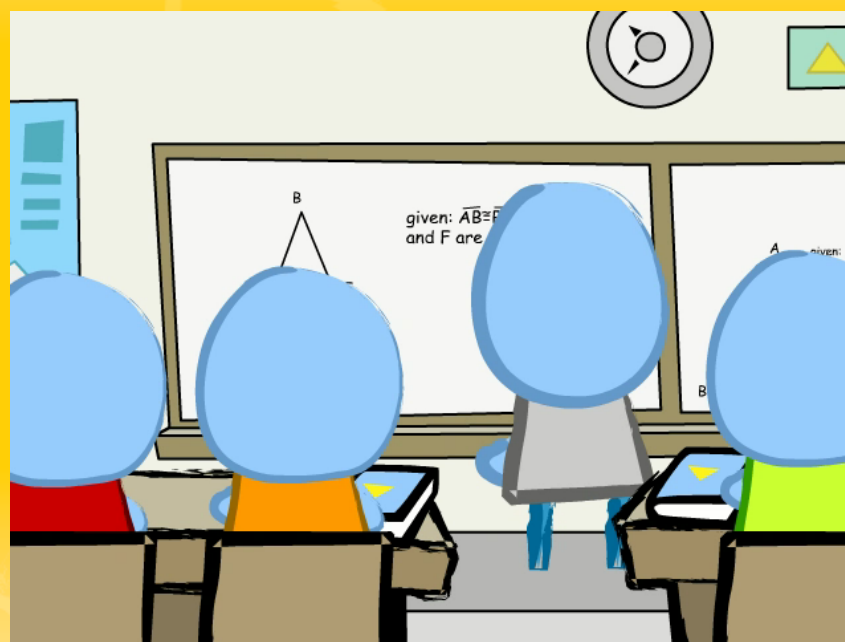
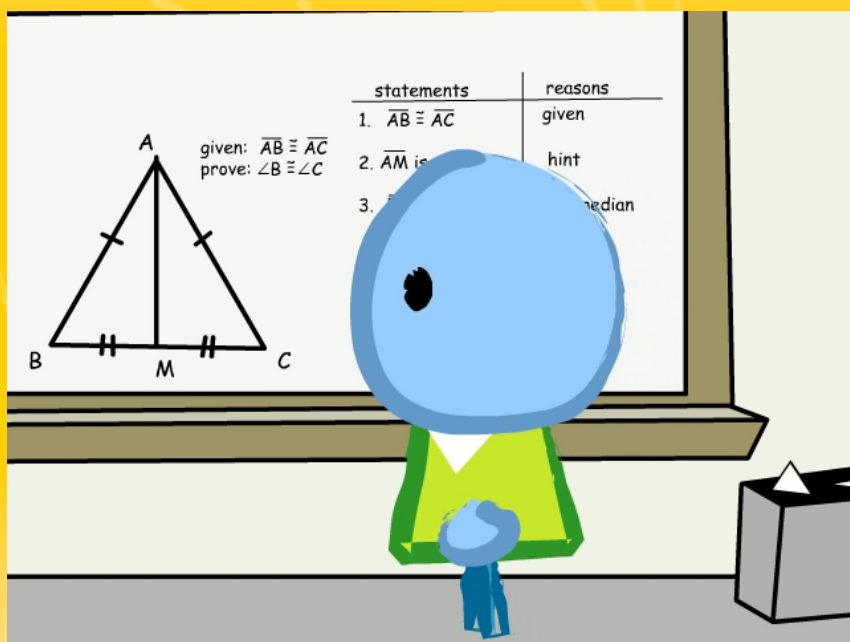
A first connection to improving instruction by way of **improving instructional systems**

- If enactment of instructional situations was studied in detail
 - E.g., by distilling the norms that make it possible for teachers and students to exchange work for knowledge...
- Those **norms** (and possibly also the **operators** or **moves** warranted by them) could be turned into **artifacts** such as rubrics or checklists
- And the use of those artifacts could be required of (suggested to) practitioners
- Like the checklists in surgery, it is expected that this move would elicit a lot of protest, but we could at least be open to **studying whether the artifacts make a difference** (in surgery they do, even in very good surgeon teams) and **developing good artifacts through research** of the kind suggested by Morris and Hiebert (2011).
 - This could be started with practitioners at the induction stage or with practitioners who have emergency certification

But in teaching we need more than **enactment** of instructional situations

- To construct opportunities to learn, particularly to give students more of a chance at using what they know to learn what they don't yet know, norms need to be broken and renegotiated.
- What knowledge do practitioners as a group have, that can support breaches of norms of instructional situations
- Let's look at one example

Another example of an instructional situation: Installing a theorem in high school geometry



The norm: Sanctioning

- For a teacher to be able to hold students accountable to use a statement as true, the **statement** needs to have been **sanctioned**, or **invested of status** (theorem, lemma, property, etc.)
 - This sanctioning is done by the teacher
- Classroom instruction (as a case of instructional systems) could be **more knowledgeable** if classrooms had **publicly displayed prompts** such as “Could a statement we proved for homework help us prove another statement later? How would it make sense to refer to or remember it?”
- But that is a relatively low bar...
- Is there **group knowledge** about mathematics teaching that could **support** handling the installation of a theorem through **letting students figure out** that the **statement proved is useful**?

When we showed the animation to experienced high school geometry teachers...

- They could **value** the animated teacher's intention of giving students **the opportunity to recognize the usefulness** of what they had proved.
- They saw as acceptable that the connection be spotted while solving the second problem, after having done the second problem the long way, or even later.
- They saw the whole class as a resource that could support that move: at least some students would see a connection and they could share this with all.
- They saw the episode as encouraging students to translate between proof problems, typically expressed in terms of a diagram, and theorems typically expressed in terms of general concepts.

In other words, they could see value in breaching the “sanctioning” norm

On account of dispositions toward

Creating context for **mathematical practices** such as making **connections** between problems, recognizing **important results**, and **translating** between different registers.

As well as enhancing the students’ **intellectual** experience and encouraging students to listen to others in the class.

Of course we also heard arguments that rebuffed the actions of the animated teacher. He could have done better.

Those dispositions that can be publicly used to justify or indict teaching actions

- Are examples of what we mean by **practical rationality** and elements of the **specific knowledge held by the group**, tacitly for the most part.
 - Elicited in the context of thought experiments such as the study groups we've had with animations like the one just shown.
- These dispositions can be **associated with the norms that they breach or support** and also with more general **professional obligations**.

The **dispositions to breach** (or negotiate compliance with) **norms** of an instructional **situation** can be defended on account of professional **obligations**

- The (secondary) mathematics teacher is obligated to
- The discipline of mathematics
 - To design and manage work that represents the ideas and practices of the discipline
- Students as individuals
 - To cater to the needs and aspirations of individual students
- Students as a group
 - To manage and support the social world of classroom interaction.
- The institution of schooling
 - To operate within the provisions and constraints of the schooling system (curriculum, schedules, calendars, assessment scales, etc.)

We'd like to propose that those **obligations** could be more prominent elements of **group knowledge**

- Specifically that they could be used to justify prescribed **alternatives** to **normative moves** in instructional situations, possibly represented in situation-specific checklists or rubrics.
- For example, consider the situation of “doing proofs,” where usually the teacher is expected to provide the “given” and the “prove”

Imagine a checklist that included not only the reminder of the move to provide the given and the prove, but also defensible alternatives, such as

- Provide the statement to prove, then ask students to propose publicly what the givens could be.
- Prompt students to compare those different givens in two ways
 - Which of the proposed ‘givens’ permits to deduce the ‘prove’?
 - Which of the proposed ‘givens’ assumes less?

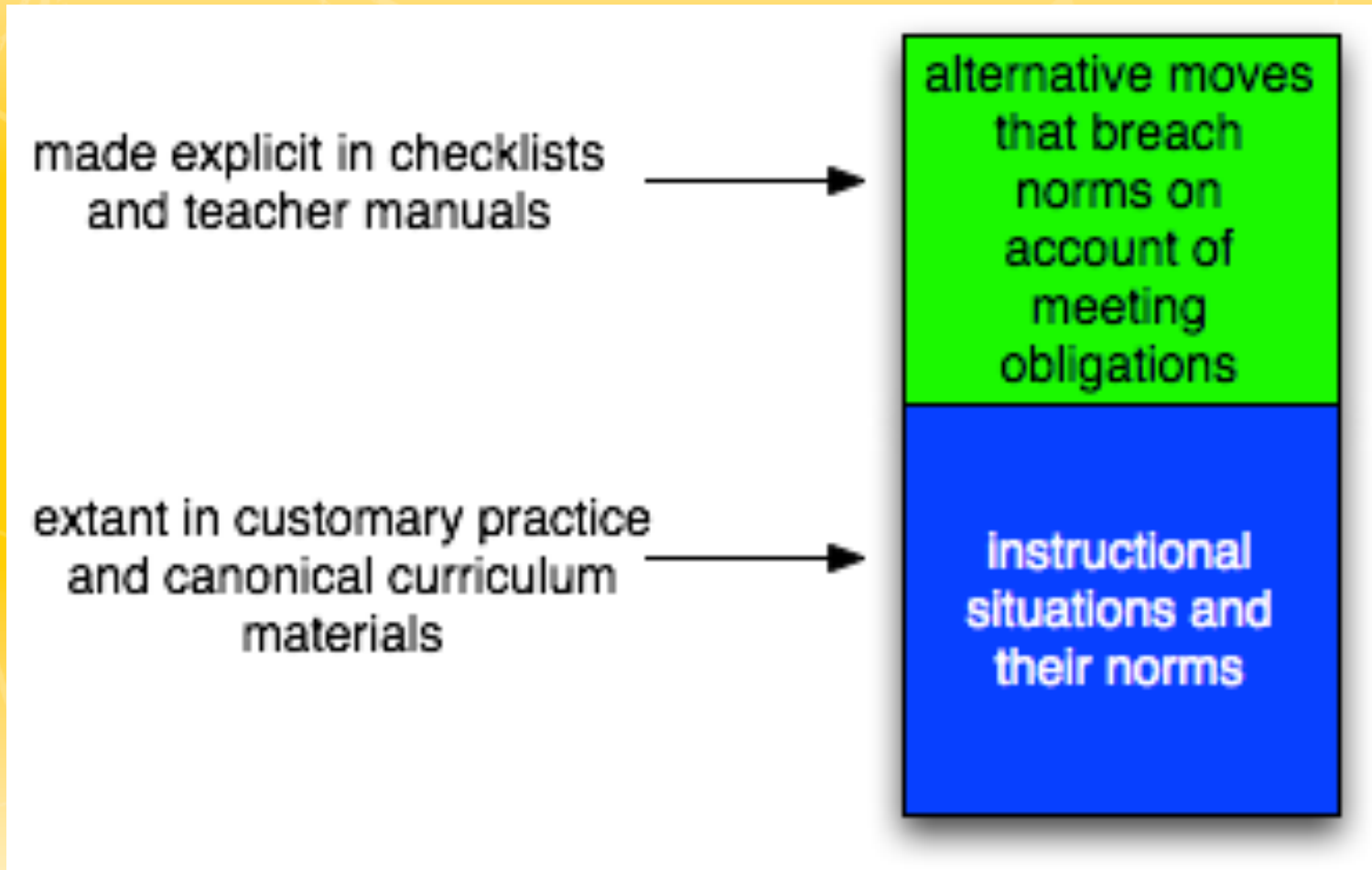
Such rubric might look like...

“Doing proofs”	Normatively	Alternative: One could...	Example
Goal	The goal to produce a proof is conveyed to students explicitly.	Ask students a question that led them to reason deductively.	If two lines look like they would intersect outside the paper, is there any way I can find out what angle they would make?
Proposition proved	The proposition is provided parsed into given and “prove.”	Provide the proposition as a concise declarative statement and have students parse it.	Prove that base angles of an isosceles triangle are congruent. Identify the givens and the prove.
Proposition proved	The proposition assigned is usually too particular to a configuration and not expected to be used.	Cast as a proof exercise a real theorem and give students an opportunity to reinvest it later.	Shown in Story 2: Isosceles triangles
Givens	Are provided to students.	Provide the “prove” statement and ask students what they’d need as givens to be able to prove it.	We want to prove that the sides of this rectangle stand in a ratio 1:2. The figure includes the midpoint of one of the longer sides and segments connecting it to the opposite vertices. Students could ask to be given that those segments make a right angle.
Prove	Is provided to students.	Provide the givens and ask students what else they could say about the objects described in the givens.	Similar problem as above, but give the right angle and ask them what could be said about that rectangle.

An approach to improvement that relies not solely on individual knowledge but also on the development and use of shared resources for teaching

- Some of those resources are curricular
 - E.g., proof problems with alternative formulations
- Other resources are instructional
 - E.g., checklists, routines, and alternative sets of moves
 - They can be developed for instructional situations so as to maintain them close to practice.
 - They can have fleshed out alternatives that support meeting the different obligations.
 - They can be improved continuously through practitioner research such as done by Gawande in surgery

Developing resources to improve collective, specific knowledge for teaching



A final thought

- On the matter of knowledge for teaching we need to find ways of making good out of the fact that teachers work in institutions, not alone
- We need to think of teacher knowledge as distributed among the knowledge in the heads of individuals and the knowledge extant in institutional artifacts and customs

References

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LessonSketch

- An online environment for teachers to create, examine, and discuss representations of practice using cartoon characters.
- www.lessonsketch.org
- The site currently contains our animations and tools for teachers to create alternatives (as slideshows of cartoon characters) and discuss them with others in forums
- Rubrics with illustrated examples will come online over time.